PCB Impedance Control:

Formulas and Resources

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Many of the formulas we use for PCB impedance calculations are readily available, but not all of them. In fact, while we are often quick to say that PCBs begin to look like transmission lines when rise times are fast enough, and perhaps toss out a few formulas for our readers, we miss at least three factors when we do so.

Not complete: It's not enough to say that Zo equals some complex formula. We may also want to know other parameters, such as propagation time, intrinsic capacitance, intrinsic inductance, and the effects of device loading and terminations. I know of no *single* source where *all* of these formulas are made available.

Too complex: Even if the formulas are available, they can be so complex that even a person with a considerable background in mathematics can't use them without an intolerable probability of arithmetic error.

Not right form: It is fine, for example, to say that Zo is some complex function of width, thickness, and height above the plane. But what if what you really want to know is what width to use to hit a given Zo? Or what height is required? Most of our readers are not prepared to tackle this type of formula manipulation.

This article will do two things: (1) bring together all the relevant formulas into one place, along with source references, and (2) show you where there are several, free resources available for dealing with them.

The formulas

Figure 1 provides the basic formulas for Zo (in Ohms), Co (in pf/in) and Tpd (ns/in) for the most common PCB trace configurations. While many sources have some of these formulas, I only know of one source (footnote 1) that contains all of them. One note, however, about embedded microstrip. The adjustment for the relative dielectric coefficient is reasonable if the material above the trace is more than 4 mils thick and is of the same dielectric coefficient as the material below the trace. If the thickness is less than that, or if the dielectric coefficient is lower than the material below the trace, the actual result will be between those calculated for microstrip and for embedded microstrip.

Note also that the embedded microstrip configuration, for infinite embedding, is the same as asymmetric stripline with one plane infinitely far away. Yet the formulas, while close, do not give identical results in this cases. This illustrates the problem with all these formulas. They are approximations and are based on simplifying assumptions. But they are only guides; none will give exact results. If exact results are really important, get your board fabricator on board early in the design process and rely on their expertise.

Almost all references point out that for a transmission line, Zo is the square root of Lo/Co. (**Figure 2a**) Therefore, the intrinsic inductance, Lo, can be calculated as shown in Figure 2b. At least one source provides the formula shown in Figure 2c for calculating the inductance of a flat trace over a plane.(footnote 2) The results of these two formulas for microstrip can differ by as much as 10%, which illustrates again that all these formulas are approximations based on simplifying assumptions, and different approaches can sometimes lead to different results.

(a) $\quad Zo \; = \; \sqrt{\dfrac{Lo}{Co}} \qquad$ Ohms

(b) $\quad Lo \; = \; Co \, * \, Zo^{\,2} \qquad$ pH/in

(c) $\quad L = 5 * Ln\left(\dfrac{2 * Pi * H}{W}\right) \qquad$ nH/in

Figure 2
Formulas for trace impedance

Devices placed along a trace add capacitance. This capacitance "loading" has an effect on both Zo and on Tpd. The correction factor is the square root of (1 + Cd/Co*l) where Cd is the sum of the capacitive loads and l is the length of the trace (**Figure 3a**). The correction factors for Zo and Tpd are shown in Figure 3b and 3c.

These adjustments are correct for parallel terminations and they are found in most references. Motorola has a complicated derivation that shows an additional effect from series termination. (Footnote 3). The theory is complex, but fundamentally involves the fact that the
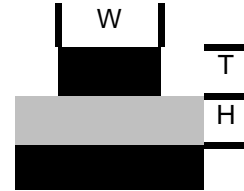

Transmission Line Formulas

Microstrip

$$Z_0 = \frac{87}{\sqrt{e_r + 1.41}} \, Ln\left(\frac{5.98 H}{.8W + T}\right)$$

$$C_0 = \frac{.67\,(e_r + 1.41)}{Ln\,[5.98\,H\,/(.8W + T)]}$$

$$t_{pd} = 1.017\,\sqrt{.475\,e_r + .67}$$

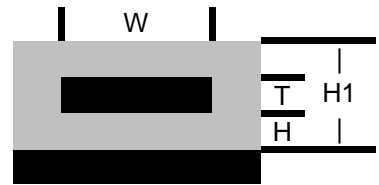## Embedded Microstrip

First, let: $\quad e_r' = e_r\left[1 - \exp\left(\frac{-1.55\,H1}{H}\right)\right]$

$$Z_0 = \frac{60}{\sqrt{e_r'}} \, Ln\left(\frac{5.98\,H}{.8W + T}\right)$$

$$C_0 = \frac{1.41\,(e_r')}{Ln\,[5.98\,H\,/(.8W + T)]}$$

$$t_{pd} = .08475\,\sqrt{e_r'}$$

Stripline

$$Z_0 = \frac{60}{\sqrt{e_r}} \, Ln\left(\frac{1.9(2H + T)}{(.8W + T)}\right)$$

$$C_0 = \frac{1.41\,e_r}{Ln\,[3.81\,H\,/(.8W + T)]}$$

$$t_{pd} = 1.017\,\sqrt{e_r}$$
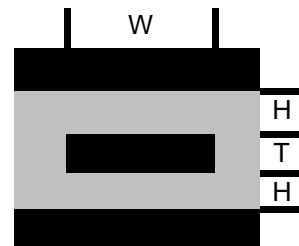
Dual or Asymmetric Stripline

$$Z_0 = \frac{80}{\sqrt{e_r}} \, Ln\left[\frac{1.9\,(2H + T)}{(.8W + T)}\right]\left[1 - \frac{H}{4\,(H1)}\right]$$

$$Z_0 = \frac{80}{\sqrt{e_r}} \, Ln\left[\frac{1.9(2H + T)}{(.8W + T)}\right]\left[1 - \frac{H}{4\,(H + C + T)}\right]$$

$$C_0 = \frac{2.82\,e_r}{Ln\,[2\,(H - T)\,/(.268\,W + .335\,T)]}$$
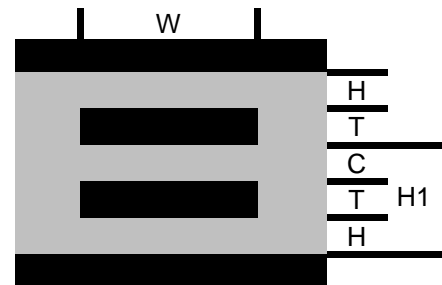
$$t_{pd} = 1.017\,\sqrt{e_r}$$

Figure 1
Formulas for impedance calculations. Units for Zo, Co, and Tpd are Ohms, Pf/in, and ns/in, respectively.

$$\sqrt{1 + \frac{Cd}{(Co * l)}}$$ (a) Correction Factor

$$Zo' = \frac{Zo}{\sqrt{1 + \frac{Cd}{Co * 1}}}$$ (b) Impact on Zo

$$t_{pd}' = t_{pd} \sqrt{1 + \frac{Cd}{(Co * l)}}$$ (c) Impact on Tpd

$$2 * \left[\sqrt{\left(1 + \frac{Cd}{Co * l}\right)} - 1 \right] + 1$$ (d) Correction Factor for Series Termination

Figure 3
Impact of Capacitive Loading

capacitive loads are charged through the series load, somewhat slowing down the rise time, and therefore the propagation time, of the trace. The adjusted correction factor for series termination is shown in Figure 3d. Note that in all these cases the adjustment factor collapses to 1.0 if Cd = 0.

Finally, the voltage reflection coefficient, ρ, of a transmission line is given by

$$\rho = (R_L - Zo)/(R_L + Zo)$$

One characteristic of a transmission line is that if it is terminated in a resistive load equal to its characteristic impedance (Zo), a signal traveling down the trace will be completely absorbed by the load and not reflect back. If the trace is left open at the far end (R_L is infinite), the signal reflects back with the same polarity and magnitude. But if the trace is *shorted* at the far end ($R_L = 0$ and therefore ρ = -1), the signal reflects back with the same magnitude but the *opposite* polarity.

The Resources

It is really not too difficult to build your own spreadsheet to make these calculations. An example is shown in **Figure 4a** and the formulas for row 14 are shown in Figure 4b.(footnote 4) These formulas can simply be copied to additional rows to show the effects of a wide range of relative dielectric coefficients. Building a spreadsheet has the added benefit that you can build graphs to illustrate various relationships, and solve for any variable (perhaps, though, through an interative process).

Barry Olney has an interactive web site where you can enter various parameters, and it will display the impedance of the various trace layers. The web site can be found at www.icd.com.au/board/board.html.

Polar Instruments, Ltd. has a freeware Windows calculator that allows you to calculate several trace parameters given the others. You can obtain this calculator at www.polar.co.uk and follow the link to the calculator. It is not readily apparent that you need to enter the data in mils or mm or significant round off error may result.

UltraCAD Design, Inc. has a freeware Windows calculator that has all the formulas contained in this article, and that can solve for virtually any parameter given the others. It also has a help file that discusses the formulas and their sources. You can obtain it at www.ultracad.com and follow the links to "calculators."

Note: The article, as published, contained illustrations of these last three resources. The illustrations have been omitted here because the pictures added almost 900k to the .pdf file!

Summary

Articles and seminars have shown you the significance, importance, and impact of transmission lines on PCBs. The formulas and their sources are summarized here, but can admittedly be difficult to use. There are, however, several freeware resources available to you on the web and elsewhere that can simplify this task. The only thing left is for you to take advantage of these resources.

Footnotes
1. IPC-D-317, April, 1990, "Design Guidelines for Electronic Packaging Utilizing High-Speed Techniques", pp. 17-25.
2. Ott, Henry, "Noise Reduction Techniques in Electronics," Wiley Interscience, 1988, p. 281
3. "MECL System Design Handbook," Rev. 1, Motorola Semiconductor Products, Inc. 1988, p. 157
4. These formulas are correct for Quattro Pro and for Lotus 1-2-3. They might need some very minor adjustments for Excel spreadsheets.

```
B14  2
C14  87*@LN(5.98*$D$5/(0.8*$D$7+$D$6))/(B14+1.41)^0.5
D14  0.67*(B14+1.41)/@LN(5.98*$D$5/(0.8*$D$7+$D$6))
E14  1.017*(0.475*B14+0.67)^0.5
F14  12/E14
G14  (C14^2)*D14/1000
I14  60*@LN((4*(2*$D$5+$D$6))/(0.67*3.14159*(0.8*$D$7+$D$6)))/((B14)^0.5)
J14  1.41*B14/@LN(3.81*$D$5/(0.8*$D$7+$D$6))
K14  1.017*(B14)^0.5
L14  12/K14
M14  (I14^2)*J14/1000
O14  80*(1-$D$5/(4*($D$5+$D$8+$D$6))))*@LN(1.9*(2*$D$5+$D$6)/(0.8*$D$7+$D$6))/(B14)^0.5
P14  2.82*B14/@LN(2*($D$5-$D$6)/(0.268*$D$7+0.335*$D$6))
```

Figure 4b
Formulas used in spreadsheet example

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1															
2	This program is set up to calculate various impedances and capacitances associated														
3	with strip lines, microstrips, etc. Enter data in inches.														
4															
5	let:	h =	0.009					Zo is in ohms							
6		t =	0.0008					Co is capacitance (pf)/inch							
7		w =	0.01					Tp is propagation delay in ns/ft							
8		c =	0.0076												
9															
10		Microstrip						Stripline						Dual Stripline	
11	dialectric	Zo	Co	Tp ns/ft	Tp in/ns	Inductance		Zo	Co	Tp ns/ft	Tp in/ns	Inductance		Zo	Co
12	coeff	(Ohms)	(pf/in)			(nH/in)						(nH/in)			
13															
14	2.0	85.317	1.262	1.294	9.270	9.183		59.446	2.073	1.438	8.343	7.327		69.002	3.286
15	2.1	84.093	1.299	1.313	9.137	9.183		58.013	2.177	1.474	8.142	7.327		67.339	3.451
16	2.2	82.920	1.336	1.332	9.010	9.183		56.679	2.281	1.508	7.955	7.327		65.791	3.615
17	2.3	81.795	1.373	1.350	8.888	9.183		55.433	2.384	1.542	7.780	7.327		64.345	3.779

Figure 4a Spreadsheet can be used for impedance calculations